Proposal for the Measurement of Bell-like Correlations from Continuous Variables

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Abstract

We show theoretically that Bell-type correlations can be observed between continuous variable measurements performed on a parametric source. An auxiliary measurement, performed on the detection environment, negates the possibility of constructing a local realistic description of these correlations.

Entanglement is a defining feature of quantum mechanical systems and leads to correlations between sub-systems of a very non-classical nature. These in turn lead to fundamentally new interactions and applications in the field of quantum information [1]. The strange nature of entanglement was first pointed out by Einstein, Podolsky and Rosen (EPR) [2] for the continuous variables of position and momentum. Though raising philosophical questions, their formulation did not lead to predictive differences between quantum mechanics and local realistic theories. An experimental demonstration of the EPR effect was made by Ou et al [3] by measuring the 2nd order correlations between the conjugate quadrature amplitudes of an optical parametric source. The quadrature amplitudes are the optical analogues of position and momentum and can be measured efficiently with homodyne detection. Although the observed correlations were shown to conflict with semi-classical optical theory they were not shown to conflict with local realistic theories in general.

The deeper mysteries of entanglement were quantified by Bell [4] in his famous inequalities. These raise testable differences between quantum mechanics and **all** local realistic theories. Numerous experimental tests of Bell-type inequalities have been made in optics starting with Aspect [5]. Violations, showing agreement with quantum mechanics, of over 100 standard deviations [6] and over large distances [7] have now been performed. In these experiments the correlations between discrete measurements of particle number are studied. These correlations are to 4th order in the optical fields.

In this letter we show how Bell-type correlations can be obtained from measurements of continuous variables. Although defined in terms of 4th order correlations we are able to express our result in terms of products of only 2nd order correlations. This work is of clear significance to the new field of continuous variable quantum information [8], but also offers new insights into the fundamental mechanism of entanglement. Unlike some earlier proposals [9] our scheme can be applied to macroscopic fields. Our proposal differs from previous macroscopic theories of this kind [10] in two ways: (i) The source upon which we base our quantum mechanical demonstration is a standard optical parametric amplifier. Previous proposals required the use of more exotic sources; and (ii) Unlike previous proposals

in which the continuous variables were discretized, we instead use the standard device of decomposition into two orthogonal polarization bases. The possibility of constructing a local realistic description of these correlations, based on the positive Wigner function that describes the parametric amplifier, can be disallowed by an auxiliary intensity measurement performed on the detection environment.

Consider the generic correlation experiment shown in Fig.1. Correlated beams of particles are emitted from a source (S) in opposite directions, A and B. Two distinct paths (p and m) are available to the particles in each beam. These could be different spatial paths or orthogonal polarizations (or spins), as in standard realizations. The two paths are combined and then spatially separated to form a different pair of orthogonal paths $+$ and $-$. The combiners, $C(\theta)$, are black boxes such that it is not possible, for a general value of the mixing parameter θ, to determine from measurements of $+$ and $-$ whether a particular particle took path p or m. We also allow for a classical phase reference (i.e. local oscillators) to be established at the measurement sites. Measurements are then made on the $+$ and $-$ paths of each beam giving results $R^{+}(\theta)$ and $R^{-}(\theta)$ respectively. In the standard case these measurements are simply the presence (1) or absence (0) of a particle in a particular path in some time interval. More generally they may represent the count rate of particles in a particular path. We allow for the most general case in which they may also be constructed from the values of some continuous properties of the particles (such as position or momentum) averaged over some time interval. We can form correlation functions of the following form

$$R^{ij}(\theta_A,\theta_B) = R^{i}_{A}(\theta_A)R^{j}_{B}(\theta_B) \tag{1}$$

where $i,j=+,-$. We then construct the normalized averages

$$P^{ij}(\theta_A,\theta_B) = \frac{\langle R^{ij}(\theta_A,\theta_B)\rangle}{\sum_{k,l=\pm}\langle R^{kl}(\theta_A,\theta_B)\rangle} \tag{2}$$

It is a well known result [11] that provided the P's have the form of probabilities (bounded between 0 and 1) then in any local realistic description the correlations will be bounded by the following Bell inequality

$$B = |E(\theta_A,\theta_B) + E(\theta'_A,\theta'_B) + E(\theta'_A,\theta_B) - E(\theta_A,\theta'_B)| \leq 2 \tag{3}$$

where

$$E(\theta_A,\theta_B) = P^{++}(\theta_A,\theta_B) + P^{--}(\theta_A,\theta_B) - P^{+-}(\theta_A,\theta_B) - P^{-+}(\theta_A,\theta_B) \tag{4}$$

The inequality of Eq. (3) can be applied in the case of the R's being constructed from continuous measurements provided $R^{ij}\geq 0$ (and thus $0\leq P^{ij}\leq 1$). It remains to be shown whether there are any particular continuous measurements which will violate this inequality for some particular quantum states.

To pursue this goal we first review the standard optical example of a state which violates this inequality with discrete measurements. Such a state is the number-polarization entangled state $\chi/\sqrt{2}\,(|1_h,1_h\rangle+|1_v,1_v\rangle)+|0\rangle$ which is approximately produced by a parametric down converter operating at low conversion efficiency ($\chi << 1$). Here $|1_i,1_j\rangle \equiv |1_i\rangle_A\otimes|1_j\rangle_B$

and 1_h and 1_v represent single photons in the horizontal and vertical polarizations respectively. The requirement of low conversion efficiency is so that higher photon number terms (which appear as products of higher powers of χ) can be neglected. In the following it will be more convenient to work in the Heisenberg picture. In this picture the action of the down converter is to evolve vacuum state, input annihilation operators $C_{h,v}$ and $D_{h,v}$ according to

$$\hat{A}_{h,v} = \hat{C}_{h,v} + \chi \hat{D}^{\dagger}_{h,v}, \quad \hat{B}_{h,v} = \hat{D}_{h,v} + \chi \hat{C}^{\dagger}_{h,v} \tag{5}$$

where as before $\chi << 1$ has been assumed. Our two paths, p and m, in this example are the horizontal ($\hat{A}_h$ and $\hat{B}_h$) and vertical ($\hat{A}_v$ and $\hat{B}_v$) polarization modes. The mixer C is then some combination of polarizing optics which decomposes our beams into a different, orthogonal polarization basis set ($+$ and $-$). This corresponds to the transformation

$$\begin{aligned} \hat{A}_{+}(\theta_A) &= \cos\theta_A \hat{A}_h + \sin\theta_A \hat{A}_v, \quad \hat{A}_{-}(\theta_A) = \cos\theta_A \hat{A}_v - \sin\theta_A \hat{A}_h \\ \hat{B}_{+}(\theta_B) &= \cos\theta_B \hat{B}_h + \sin\theta_B \hat{B}_v, \quad \hat{B}_{-}(\theta_B) = \cos\theta_B \hat{B}_v - \sin\theta_B \hat{B}_h \end{aligned} \tag{6}$$

Photon counting is then performed on the beams and we define

$$\begin{aligned} R^i_A(\theta_A) &= \hat{A}^{\dagger}_i(\theta_A)\hat{A}_i(\theta_A) \\ R^i_B(\theta_B) &= \hat{B}^{\dagger}_i(\theta_B)\hat{B}_i(\theta_B) \end{aligned} \tag{7}$$

with $i = +, -$. The definitions then follow as per Eqs. (1), and (2). An explicit calculation gives the result

$$E(\theta_A, \theta_B) = \cos 2(\theta_A - \theta_B) \tag{8}$$

where terms of order higher than χ^2 have been neglected. Choosing the angles $\theta_A = 3\pi/8$, $\theta'_A = \pi/8$, $\theta_B = \pi/4$, $\theta'_B = 0$ we find $B = 2\sqrt{2}$, a clear violation of Eq.(3).

We now consider how we might decompose this result into continuous variable measurements. The quantum mechanical properties of correlation functions such as

$$R^{++} = \langle \hat{A}^{\dagger}_{+}(\theta_A)\hat{A}_{+}(\theta_A)\hat{B}^{\dagger}_{+}(\theta_B)\hat{B}_{+}(\theta_B)\rangle \tag{9}$$

are at the heart of our result. However, an arbitrary field operator, $\hat{F}$, can be written as a sum of the conjugate in-phase, $\hat{X}_{F;1} = \hat{F} + \hat{F}^{\dagger}$, and out-of-phase, $\hat{X}_{F;2} = i(\hat{F} - \hat{F}^{\dagger})$, quadrature amplitude operators via $\hat{F} = 1/2(\hat{X}_{F;1} - i\hat{X}_{F;2})$. As noted earlier these operators represent continuous variable observables which can be measured via homodyne detection with respect to local oscillator fields. Thus we can write Eq.(9) in the form

$$\begin{aligned} R^{++} = \frac{1}{16}\langle &(\hat{X}^{+}_{A;1}(\theta_A) + i\hat{X}^{+}_{A;2}(\theta_A))(\hat{X}^{+}_{A;1}(\theta_A) - i\hat{X}^{+}_{A;2}(\theta_A)) \\ &\times(\hat{X}^{+}_{B;1}(\theta_B) + i\hat{X}^{+}_{B;2}(\theta_B))(\hat{X}^{+}_{B;1}(\theta_B) - i\hat{X}^{+}_{B;2}(\theta_B))\rangle \end{aligned} \tag{10}$$

This in turn can be expanded to give the second order correlation function

$$\begin{aligned} R^{++} = \frac{1}{16}\Big(&2(V^{+}_{A;1,B;1})^2 + 2(V^{+}_{A;2,B;2})^2 + 2(V^{+}_{A;2,B;1})^2 + 2(V^{+}_{A;1,B;2})^2 + \\ &V^{+}_{A;1}V^{+}_{B;1} + V^{+}_{A;2}V^{+}_{B;2} + V^{+}_{A;2}V^{+}_{B;1} + V^{+}_{A;1}V^{+}_{B;2} - \\ &(1/i)[\hat{X}^{+}_{A;1}, \hat{X}^{+}_{A;2}](V^{+}_{B;1} + V^{+}_{B;2}) - (1/i)[\hat{X}^{+}_{B;1}, \hat{X}^{+}_{B;2}](V^{+}_{A;1} + V^{+}_{A;2}) - \\ &[\hat{X}^{+}_{A;1}, \hat{X}^{+}_{A;2}][\hat{X}^{+}_{B;1}, \hat{X}^{+}_{B;2}]\Big) \end{aligned} \tag{11}$$

where we have assumed Gaussian noise statistics (a valid assumption for parametric amplification) and thus expanded 4th order correlations via $\langle (X_i X_j)^2 \rangle = \langle (X_i)^2 \rangle \langle (X_j)^2 \rangle + 2\langle (X_i X_j) \rangle^2$ and defined $\langle (X_i X_j) \rangle = V_{i,j}$ and $\langle (X_i)^2 \rangle = V_i$. The other correlation functions (R^{--}, R^{+-}, R^{-+}) can be formed in a similar way and hence $E(\theta_A, \theta_B)$ constructed. The terms in the 1st line of Eq.(11) are the 4-mode equivalent (2 spatial $\times$ 2 polarization) of the 2-mode correlations measured by Ou et al [3]. These produce the cosine dependence on polarizer angle seen in $E(\theta_A, \theta_B)$ (Eq.8). The 2nd line terms represent polarization independent noise which reduces the polarization visibility. The final terms are purely quantum mechanical, being products with the commutators $[\hat{X}_{k;1}, \hat{X}_{k;2}] = 2i$ ($k = A, B$). For the down converter these final terms cancel the 2nd line terms leaving high polarization visibility, as required to violate the Bell inequality.

Intriguing as this result is Eq.(11) does not actually constitute a continuous variable Bell test as it stands. To do this we must propose continuous variable measurement protocols, R_A^+ and R_B^+ from which an $R^{++} = \langle R_A^+ R_B^+ \rangle$ can be formed which is equivalent to Eq.(11). Consider the following measurement protocol. The observers at A and B prearrange synchronized time windows in which they will make their measurements. They do not prearrange what measurements they will make in a particular window. The observers randomly swap between "bright" measurements of either the in-phase or out-of-phase quadratures and "dark noise" measurements obtained by blocking the signal input and allowing *no* light to the reach the homodyne detectors. When the data thus collected is brought together the following correlation function can be formed

$$\begin{aligned} R^{++} = \langle (& (X_{A;1}^+)^2 - (X_{va;1}^+)^2 + (X_{A;2}^+)^2 - (X_{va;2}^+)^2) \\ \times \quad & ((X_{B;1}^+)^2 - (X_{vb;1}^+)^2 + (X_{B;2}^+)^2 - (X_{vb:2}^+)^2) \rangle \end{aligned} \tag{12}$$

where the $X_{vi;j}$ represent the dark noise at the two sites ($i = a, b$) and on the two quadratures ($j = 1, 2$). Our protocol thus consists of making a series of homodyne measurements, swapping between the two quadratures, each of which is then "zeroed" by subtracting off the dark noise of the measurement apparatus. Importantly, for sufficiently long data runs, the amount of redundant information will be negligible, i.e. *all* the data is used in forming the correlation function. Similar measurements are made on the minus port and the polarization angle is also randomly swapped. By using the Gaussian properties again we find Eq.12 is equivalent to

$$\begin{aligned} R^{++} = \frac{1}{16} \Big(& 2(V_{A;1,B;1}^+)^2 + 2(V_{A;2,B;2}^+)^2 + 2(V_{A;2,B;1}^+)^2 + 2(V_{A;1,B;2}^+)^2 + \\ & V_{A;1}^+ V_{B;1}^+ + V_{A;2}^+ V_{B;2}^+ + V_{A;2}^+ V_{B;1}^+ + V_{A;1}^+ V_{B;2}^+ - \\ & 2V_v(V_{B;1}^+ + V_{B;2}^+) - 2V_v(V_{A;1}^+ + V_{A;2}^+) + 4V_v^2 \Big) \end{aligned} \tag{13}$$

Eqs.(12) and (13) are the key results of this letter. They can be used to construct a test of local realistic theories based on continuous variable measurements (following the recipe of Eqs.(2) and (3)). Furthermore it can be shown that Eqs.(11) and (13) are numerically identical. Thus our continuous variable inequality will be violated by the down converter. The purely quantum mechanical terms are now those multiplied by the dark noise (V_v). For an ideal classical system this will be zero, leading to low visibility. If dark noise is present it will also affect the "bright" measurements, ensuring no violation of the Bell inequality. For

an ideal quantum mechanical system the dark noise is produced by vacuum fluctuations (as a result of non-zero commutation) and will always be at the quantum noise level ($V_v = 1$). However for the bright measurements the vacuum noise becomes correlated in the quantum mechanical case. It is indeed this ability of entangled states to correlate the environmental noise which leads to the violation of the predictions of local realistic theories. It is clear in this formulation that it is this lack of "realism" in the detection process which leads to the Bell violation. The distribution of the correlations themselves occurs in a purely local way [12].

It can be shown [13] that the correlation function of Eq.(13) is formally equivalent to that obtained from the discrete measurement $R_A^+ = \hat{A}^\dagger \hat{A} - \hat{V}^\dagger \hat{V}$ where $\hat{V}$ is the background vacuum mode. Clearly the positivity condition on R_A^+ is preserved provided $\hat{V}$ is truly a vacuum mode. Thus an essential requirement for the validity of this test is that the background measurements are truly "dark". This could be ensured by making a sensitive measurement of the light intensity entering the homodyne detectors when the signal is blocked. If we make the reasonable assumption that any stray light will be incoherent, then a dark photon number satisfying $n_{dark} << \sqrt{n_{LO}}$ where n_{LO} is the photon number of the local oscillator used to make the homodyne measurements, can be considered zero. This auxiliary measurement on the dark input prevents the construction of local hidden variable theories based on the positive Wigner function which describes this system. Any hidden variable theory which could successfully mimic the quadrature correlations would be incompatible with the observation of zero dark port intensity.

To this point we have demonstrated a new continuous variable method of measuring non-classical correlations that have already been shown to exist. We now indicate how our result can be extended to cover a class of inputs for which local realistic violations have not previously been demonstrated. Consider the arrangement of standard bright squeezed light sources shown in Fig.2. Four optical parametric amplifiers are seeded by four phase locked, horizontally polarized laser beams. The output beams will be squeezed at rf frequencies high enough such that technical noise on the laser beams can be neglected but low enough to be within the bandwidth of the amplifiers. Within this range of frequencies the quantum fluctuations of the output beams from the squeezers can be described in Fourier space by the zero-point operators

$$\delta f_i = \sqrt{G}\delta g_i + \sqrt{G-1}\delta g_i^\dagger \tag{14}$$

where G is the parametric gain of the amplifiers and δg_i's are the fluctuations associated with the input beams. These are assumed to be at the vacuum level for these frequencies. Fourier space is indicated by the lack of circumflex. These beams are then combined in the manner shown in Fig.2 to produce 4-mode squeezed beams (2 spatial $\times$ 2 polarization). The output beams can be written

$$\begin{aligned} \delta a_{h,v} &= \sqrt{G}\delta c_{h,v} + \sqrt{G-1}\delta d_{h,v}^\dagger \\ \delta b_{h,v} &= \sqrt{G}\delta d_{h,v} + \sqrt{G-1}\delta c_{h,v}^\dagger \end{aligned} \tag{15}$$

where $\delta c_{h,v} = 1/\sqrt{2}(\delta g_1 + i\delta g_2)$ and $\delta d_{h,v} = 1/\sqrt{2}(\delta g_1 - i\delta g_2)$. For low levels of parametric gain we can set $G \approx 1$ and $\sqrt{G-1} = \chi << 1$. Eqs.(5) and (15) are then formally equivalent although describing physically very different situations. On one hand Eq.(5) describes the

properties of a very low photon number light beam in the time domain. On the other hand Eq.(15) describes the small fluctuations of a macroscopic light field in the Fourier domain. These differences don't prevent us constructing Fourier domain correlation functions completely analogous to those discussed previously for the down conversion source. Thus the V's that appear in Eq.(13) are now interpreted as quadrature spectral variances measured at some rf frequency. In this way it would be possible to demonstrate correlations between the quadrature amplitude fluctuations of macroscopic light fields which violate local realistic theory predictions, a quite remarkable result.

Although the experiment just described is technically challenging it is certainly within the capabilities of present technology. Note that if the parametric gain G becomes too large then higher order terms will become important and will wash out the non-classical correlations, i.e. the effect is diluted by too much squeezing. This is also typical of the photon number correlations [14]. In Fig.3 we plot the decrease in the maximum value of B as a function of increasing squeezing. The trade-off with small levels of squeezing is that the signal to noise becomes very small, making source stability a critical factor.

We have proposed a method for observing Bell correlations with continuous variables. We suggest that a Bell inequality violation should be observable between spatially separated quadrature fluctuations of a bright source, constructed in a straightforward manner from squeezed light beams. This research represents a significant step down the path to realizing all analogues to discrete quantum information manipulations in continuous variable systems.

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FIGURES

FIG. 1. Schematic of a generic Bell experiment. See text for details

FIG. 2. Schematic of system for producing polarization/field entangled light. Here the beams δf_1 and δf_2 are combined with a $\pi/2$ phase shift on an asymmetric unpolarizing beamsplitter to produce the outputs $\delta a_1 = 1/\sqrt{2}(\delta f_1 + i\delta f_2)$ and $\delta b_1 = 1/\sqrt{2}(\delta f_1 - i\delta f_2)$. The outputs f_3 and f_4 are combined in a similar way forming outputs δa_2 and δb_2. These latter outputs are then rotated with half-wave plates into vertical polarization. The beams δa_1 and δa_2 are then combined on a polarizing beamsplitter such that they form the two polarization modes of a single beam. Similarly for δb_1 and δb_2.

FIG. 3. Plot of the maximum value of B versus the percentage squeezing. A violation is achieved for $B > 2$.

Figure 1: Ralph et al

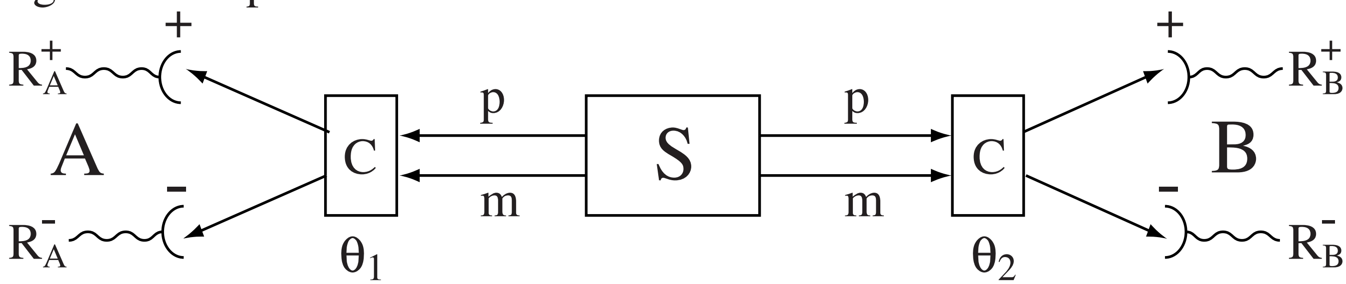

Figure 2: Ralph et al

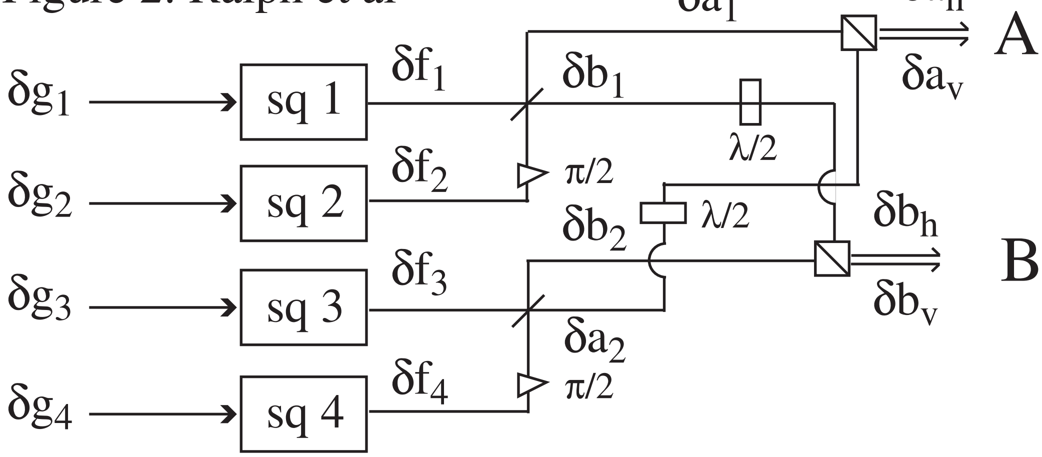

Figure 3: Ralph et al